



Elastic properties of $\text{Al}_x\text{CrMnFeCoNi}$ high-entropy alloys from *ab initio* theory

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High-entropy alloys (HEAs)

- Definition: HEAs = A + B + C + D + E + ... && $5\% < A \setminus B \setminus C \setminus D \setminus E \setminus \dots < 35\%$
- Excellent mechanical properties (e.g. CrMnFeCoNi)
- Alloying effect $\text{Al}_x\text{CrMnFeCoNi}$

EMTO-CPA

Single Crystal Elastic Constants

$$c_{ijkl} = \frac{1}{V} \frac{\partial E}{\partial e_{ij} \partial e_{kl}}$$



$$C' \quad C_{44}$$

$$C' = \frac{C_{11} - C_{12}}{2}$$

$$B(V) = V \frac{\partial^2 E(V)}{\partial V^2}$$



$$B$$

$$B = \frac{C_{11} + 2C_{12}}{3}$$

Introduction

Polycrystalline Elastic Constants

$$E = 9BG/(3B+G)$$

$$\nu = (3B-2G)/(6B+2G)$$

Voigt

$$B_V = \frac{C_{11} + 2C_{12}}{3}$$

$$G_V = \frac{C_{11} - C_{12} + 3C_{44}}{5}$$

Reuss

$$B_R = \frac{C_{11} + 2C_{12}}{3}$$

$$G_R = \frac{5(C_{11} - C_{12})C_{44}}{4C_{44} + 3(C_{11} - C_{12})}$$

Hill

$$B_H = \frac{1}{2}(B_V + B_R)$$

$$G_H = \frac{1}{2}(G_V + G_R)$$

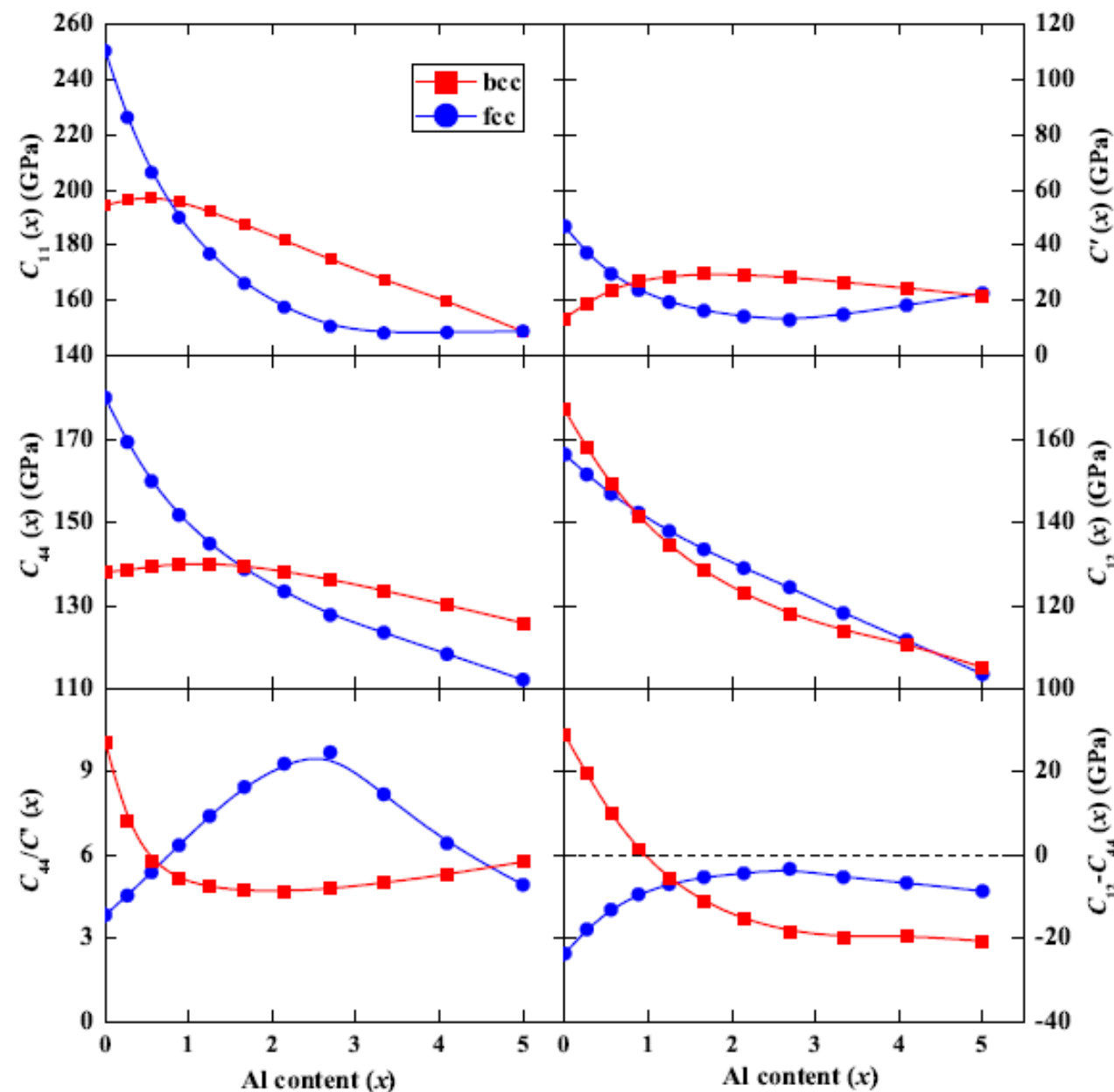
Results

Single Crystal Elastic Constants

$\text{Al}_x\text{CrMnFeCoNi}$

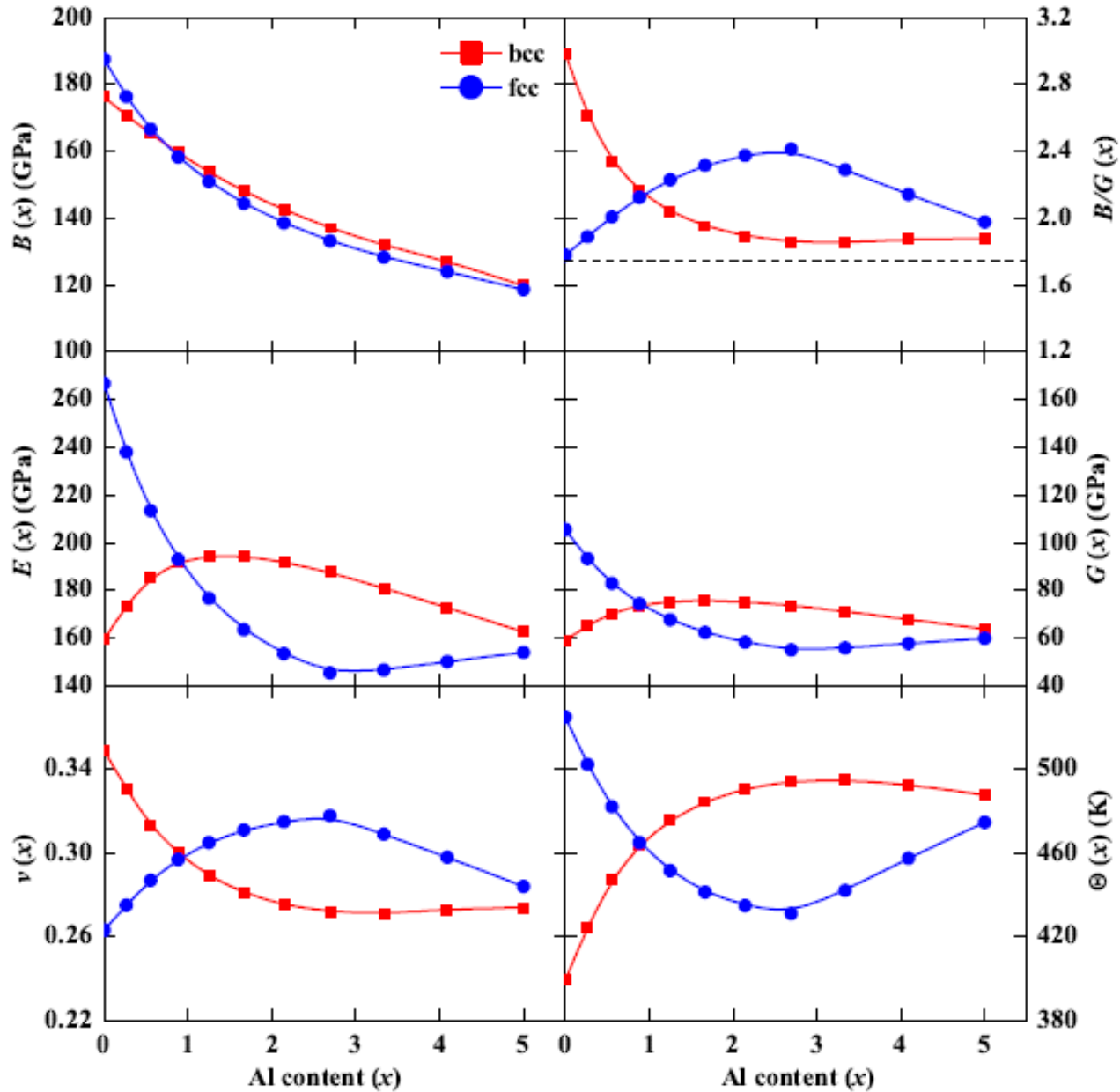
PBE-PBE

Soft-core

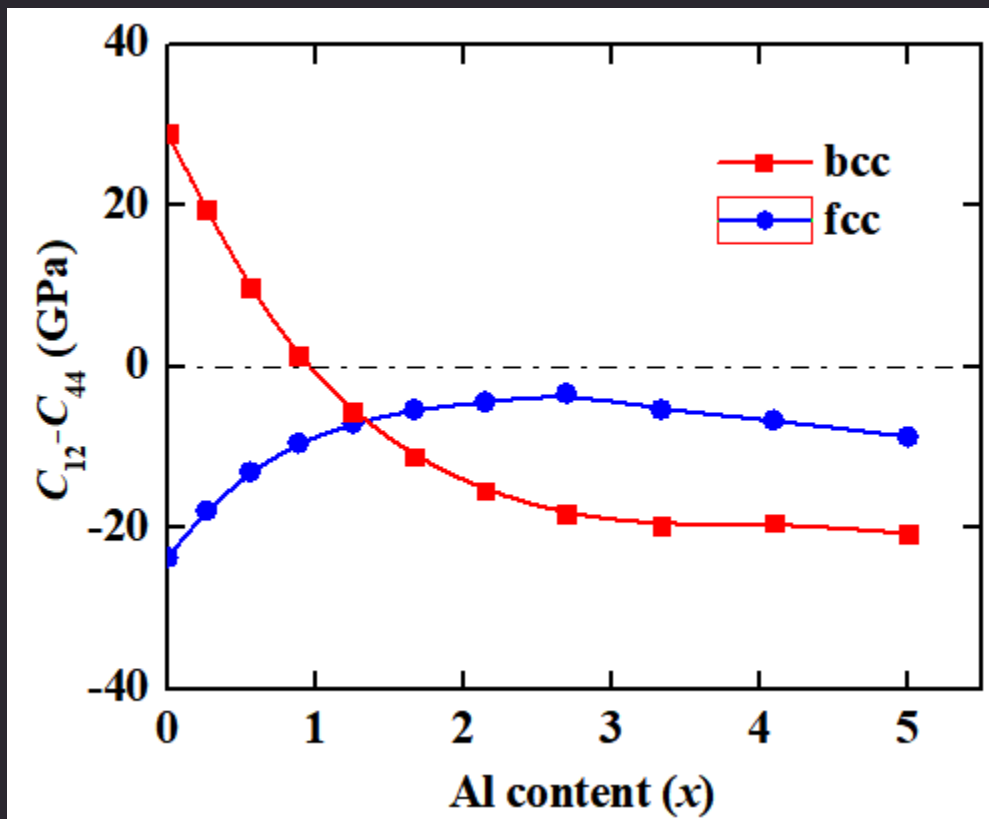


Polycrystalline Elastic Constants

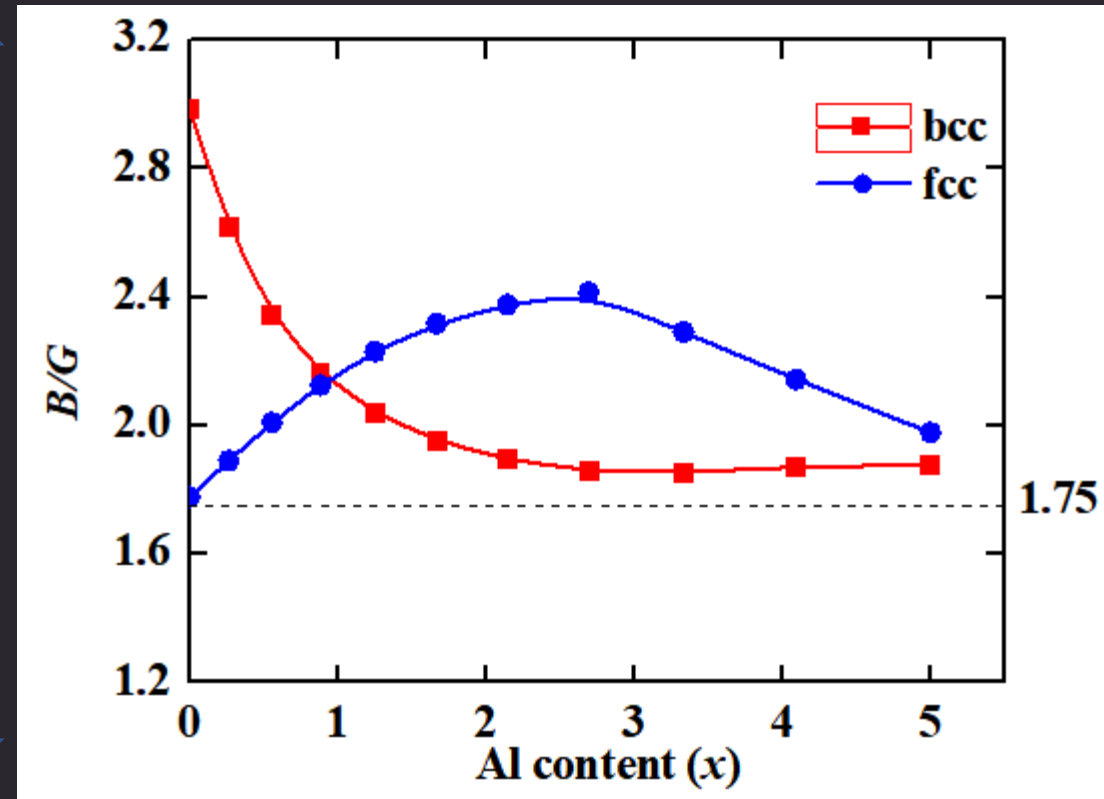
$\text{Al}_x\text{CrMnFeCoNi}$



Discussion



Cauchy pressure $CP=C_{12}-C_{44}$



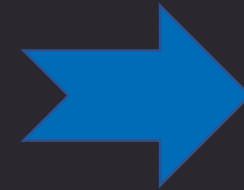
Pugh ratio B/G

Discussion

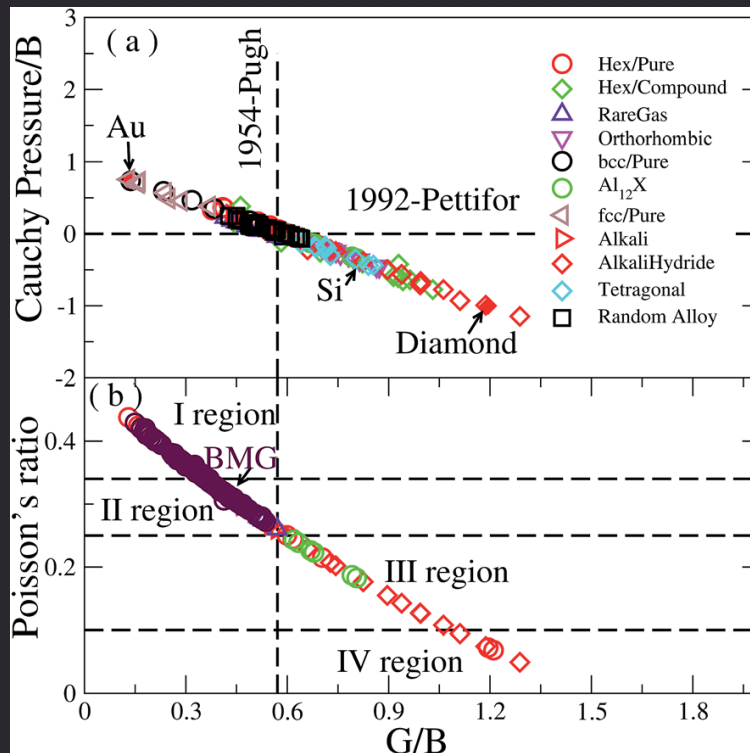
Pugh ratio B/G

Cauchy pressure $CP=C_{12}-C_{44}$

Voigt $\frac{B_V}{G_V} = \frac{C_{11} + 2C_{12}}{3} \frac{5}{C_{11} - C_{12} + 3C_{44}}$



$C_{12}-C_{44}=B_V-5/3G_V$



CP/B as a function of G/B

(Wang et al. RSC Adv., 2016, 6, 44561)

$CP= 0.995B-1.620G$

Discussion

Empirical

Cauchy pressure ($CP_0=0$)

Pugh ratio criterion ($Pugh_0=1.75$)

Voigt

$$C_{12}-C_{44}=B_V-5/3G_V$$

$CP_0=0$

$Pugh_0=1.67$

Reuss

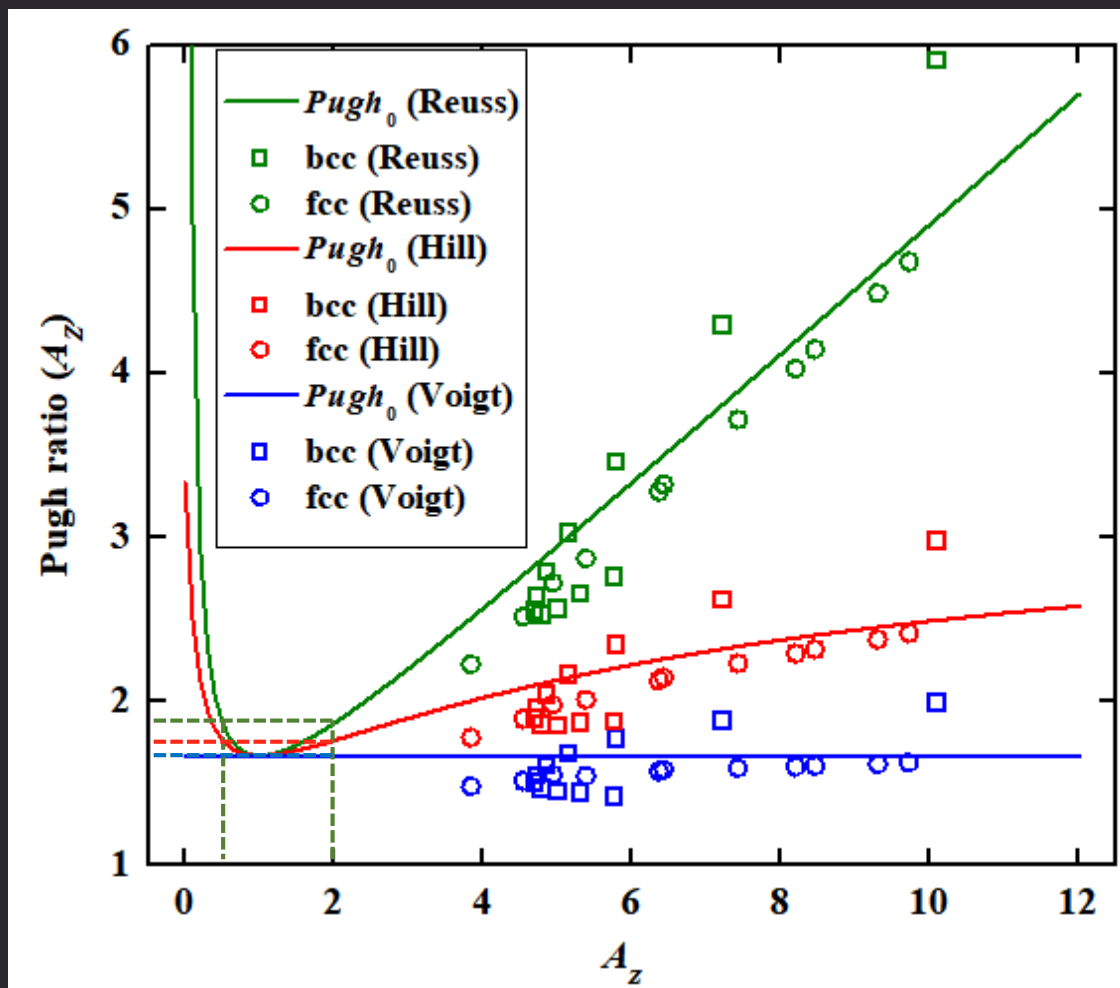
$$Pugh_0^R(A) = \frac{1}{15} \left(\frac{6}{A} + 13 + 6A \right)$$

Zener anisotropy $A=C_{44}/C'$

Hill

$$Pugh_0^H(A) = \frac{10}{3} \frac{1}{\frac{25}{\frac{6}{A} + 13 + 6A} + 1}$$

Discussion



$Pugh_0$ as a function of A_z
 $(C_{12} - C_{44} = 0)$

Alloy	CP	B/G	AZ
Fe	35.2	2.1	1.4
CrMoTiV	61.3	2.46	0.73
MoNbTiV	60.9	2.54	0.9
Fe _{0.97} Mn _{0.03}	12.01	1.82	1.42
Fe _{0.9} Mn _{0.1}	-6.71	1.64	1.55
Fe _{0.97} Ni _{0.03}	34.4	2.09	1.36
Fe _{0.9} Ni _{0.1}	55.9	2.43	1.83

Phys. Rev. B 79 (2009) 224201

Comput. Mater. Sci. 128 (2017) 185–190

Phys. Rev. B 91 (2015) 224203

Discussion

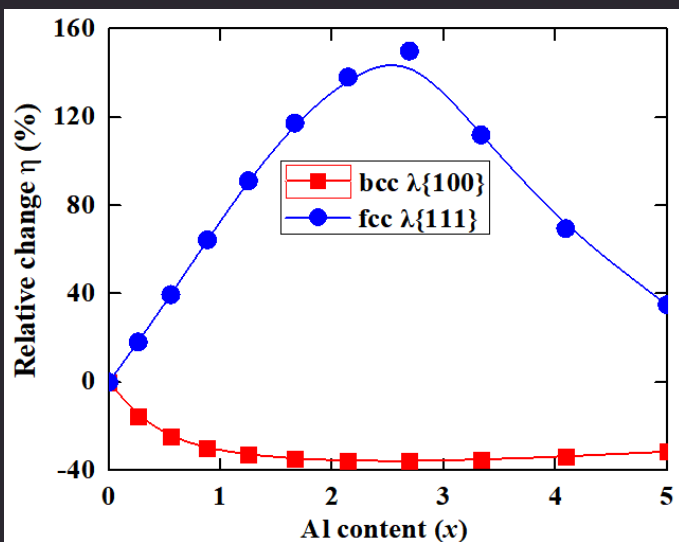
Theoretical cleavage stress

$$\sigma_{cl.}\{lmn\} = \left(\frac{E_{lmn} \gamma_{lmn}}{d_{lmn}} \right)^{1/2}$$

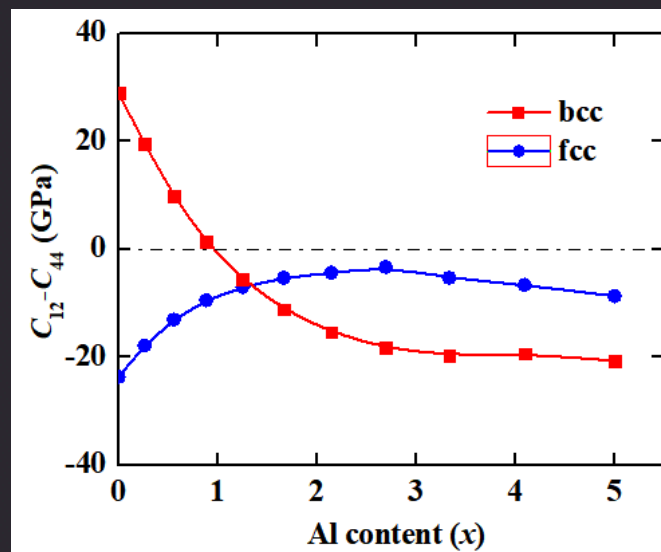
$$\text{bcc } \lambda\{110\} \equiv \sigma_{cl.}\{110\} / G\{110\}\langle 111 \rangle$$

$$\text{fcc } \lambda\{111\} \equiv \sigma_{cl.}\{111\} / G\{111\}\langle 112 \rangle$$

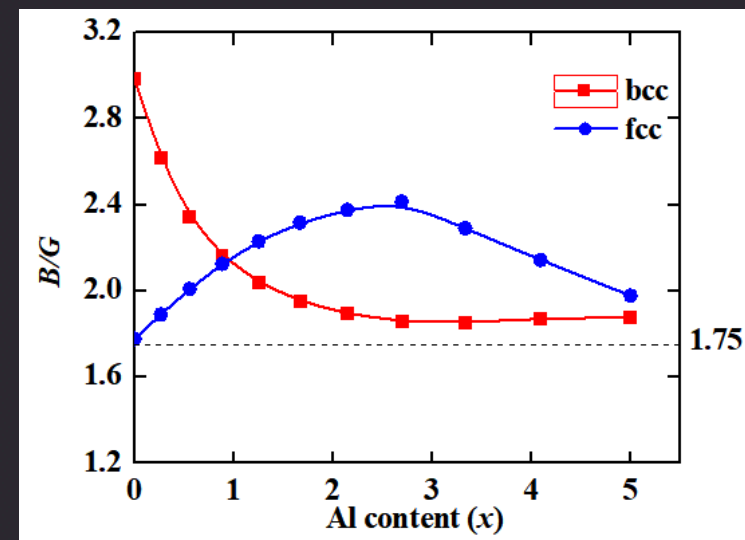
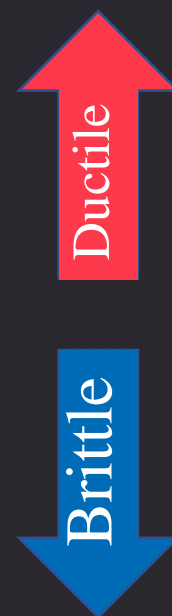
(Phys. Rev. B 91 (2015) 224203)



$$\eta = (\lambda(x) - \lambda(0)) / \lambda(0)$$

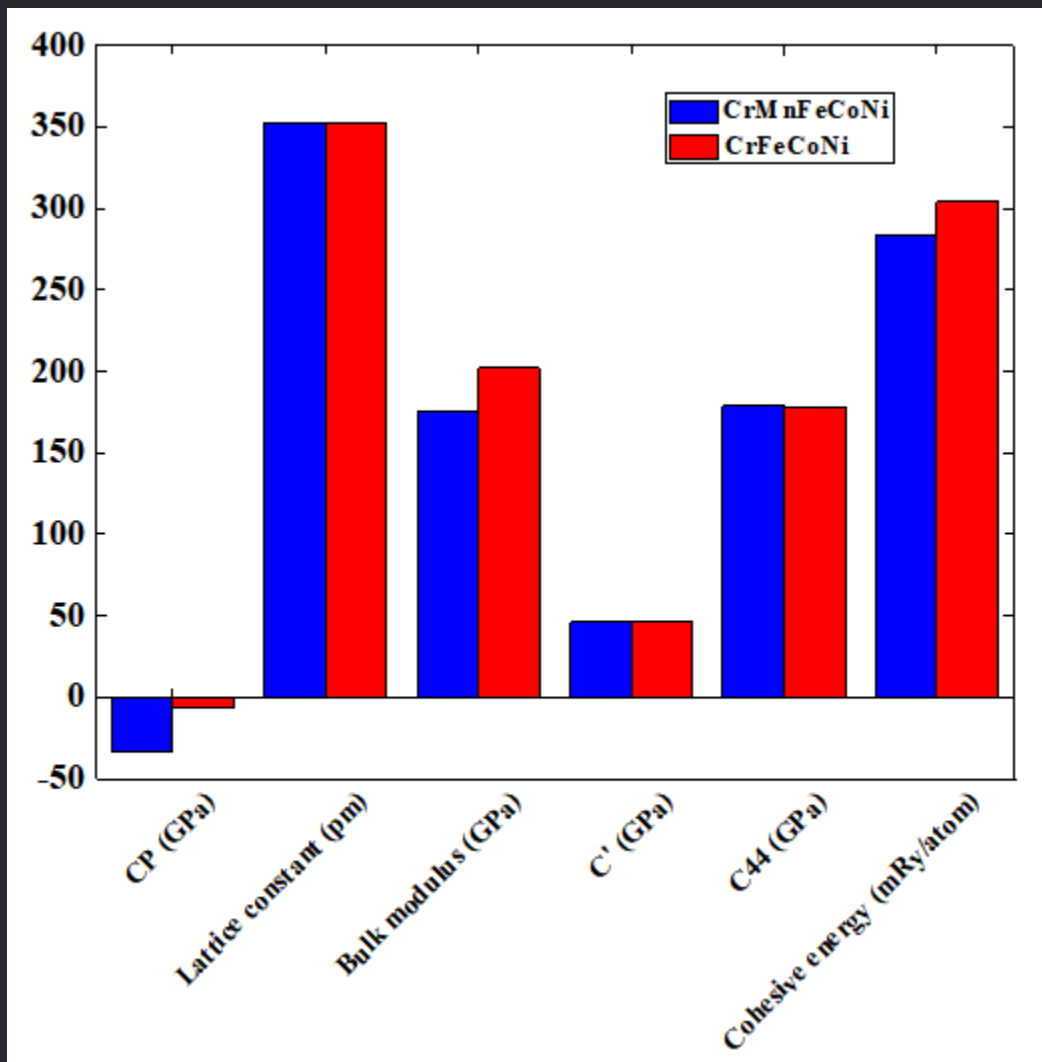


$$\text{Cauchy pressure } CP = C_{12} - C_{44}$$



$$\text{Pugh ratio } B/G$$

Discussion



$$CP \equiv B - C_{44} - \frac{2}{3} C'$$

CrMnFeCoNi

CrFeCoNi

Summary

Using *ab initio* calculations, we provide the elastic constants of paramagnetic $\text{Al}_x\text{CrMnFeCoNi}$ ($0 \leq x \leq 5$) high-entropy alloys (HEAs) in both body-centered cubic (bcc) and face-centered cubic (fcc) structures

The brittle/ductile transitions formulated in terms of Cauchy pressure and Pugh ratio become consistent only when the strong elastic anisotropy is accounted for.

The negative Cauchy pressure of CrMnFeCoNi is due to the relatively low bulk modulus and C_{12} elastic constant, which in turn are consistent with the relatively low cohesive energy.



Thanks for your attention !

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